AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Statistics 3

Monday 26 June 2017 Afternoon Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 A machine fills bags with rock salt to be used on paths and driveways.

The weight of salt in a full bag may be modelled by a normal distribution with a mean of μ kilograms and a standard deviation of 330 grams.

Calculate the sample size necessary in order that a 98% confidence interval for μ has a width of at most 200 grams. Give your answer to the nearest 10.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

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2 A council wishes to estimate the proportion of library customers who own a hand-held electronic device for reading books.

A sample of 440 customers showed that 77 owned such a device.

Stating a necessary assumption about the sample, construct an approximate 99% confidence interval for the proportion of library customers who own a hand-held electronic device for reading books.

[6 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

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3 The number of faulty peel & seal envelopes in a pack of 1000 envelopes produced by a particular company's machine may be modelled by a Poisson distribution.

Prior to a minor upgrade of the machine, the average number of faulty envelopes per pack was 8 envelopes.

(a) Following this upgrade, a randomly selected pack of 1000 envelopes contained 5 envelopes that were faulty.

Use an exact test and the 5% level of significance to investigate the success or otherwise of the minor upgrade in reducing the average number of faulty envelopes per pack.

[5 marks]

(b) The company subsequently decided to carry out a major refurbishment of the machine.

Following this refurbishment, a random sample of 50 packs of 1000 envelopes contained a total of 348 envelopes that were faulty.

Investigate, at the 1% level of significance, whether the major refurbishment of the machine has reduced the average number of faulty envelopes per pack.

[5 marks]

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4 Faults which occur in a particular make of dishwasher can be categorised as mechanical (event M), electrical (event E) or water (event W).

It has been established that $\mathrm{P}(M) = 0.45$, $\mathrm{P}(E) = 0.25$ and $\mathrm{P}(W) = 0.30$.

When a fault occurs, a dishwasher display shows one of three fault codes: $C1$, $C2$ or $C3$.

It is known that:

$$\mathrm{P}(C1 \mid M) = 0.80 \quad \mathrm{P}(C2 \mid M) = 0.05 \quad \mathrm{P}(C3 \mid M) = 0.15$$

$$\mathrm{P}(C1 \mid E) = 0.10 \quad \mathrm{P}(C2 \mid E) = 0.85 \quad \mathrm{P}(C3 \mid E) = 0.05$$

$$\mathrm{P}(C1 \mid W) = 0.00 \quad \mathrm{P}(C2 \mid W) = 0.25 \quad \mathrm{P}(C3 \mid W) = 0.75$$

A dishwasher of this particular make is showing a fault code on its display.
Calculate the probability that this dishwasher:

(a) displays fault code $C1$;

(b) has a mechanical fault, given that it displays code $C1$;

(c) has an electrical fault, given that it displays code $C2$;

(d) does not have a water fault, given that it does not display code $C3$.

[10 marks]

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5 The numbers of cars, X, and the numbers of bicycles, Y, owned by households in a town may be modelled by the following bivariate probability distribution.

		Number of cars (X)				
		0	**1**	**2**	**3**	**P($Y = y$)**
Number of bicycles (Y)	**0**	0.07	0.12	0.18	0.13	0.50
	1	0.03	0.18	0.07	0.02	0.30
	2	0.02	0.03	0.05	0.00	0.10
	3	0.03	0.02	0.00	0.00	0.05
	4	0.00	0.05	0.00	0.00	0.05
	P($X = x$)	0.15	0.40	0.30	0.15	1.00

(a) (i) Calculate exact values for $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

[4 marks]

(ii) Given that

$$\mathrm{E}(Y) = 0.85\ ,\ \mathrm{E}(Y^2) = 1.95 \text{ and } \mathrm{E}(XY) = 0.90$$

calculate exact values for $\mathrm{Var}(Y)$ and $\mathrm{Cov}(X, Y)$.

[3 marks]

(iii) Hence calculate the value of the correlation coefficient between X and Y.

[2 marks]

(b) Calculate values for the mean and the variance of:

(i) $T = X + Y$;

(ii) $D = X - Y$.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

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6 (a) The random variable X has a binomial distribution with parameters n and p.

Prove that $\mathrm{E}(X) = np$.

[3 marks]

(b) The random variable Y has a Poisson distribution with mean λ.

Prove that $\mathrm{Var}(Y) = \lambda$.

[3 marks]

(c) A company manufactures metal screws. The probability that a screw is faulty is 0.005. The screws are sold in packets and boxes. The screws in a packet or in a box may be considered to be a random sample.

(i) Customer A buys a packet of 50 screws.

Determine the probability that the packet contains exactly one faulty screw.

[2 marks]

(ii) Customer B buys a small box of 250 screws.

Use a distributional approximation to estimate the probability that the small box contains fewer than two faulty screws.

[3 marks]

(iii) Wholesaler C buys 50 large boxes, each containing exactly 1000 screws.

Use a distributional approximation to estimate the probability that the **total** number of faulty screws in the 50 large boxes is more than 240.

[5 marks]

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7 Results from an investigation into the characteristics of saltwater crocodiles included the following summarised data on the lengths, x metres, of a sample of 40 adult males and the lengths, y metres, of a sample of 30 adult females.

Males: $\sum x = 181.20$ $\sum (x - \bar{x})^2 = 11.7022$

Females: $\sum y = 86.40$ $\sum (y - \bar{y})^2 = 3.4806$

(a) Investigate, at the 5% level of significance, the hypothesis that the mean length of adult male saltwater crocodiles exceeds that of adult female saltwater crocodiles by more than 1.5 metres.

[9 marks]

(b) Deduce that, for the test of the hypothesis in part **(a)**, the critical value of $\bar{X} - \bar{Y}$ is 1.68 , correct to three significant figures.

[2 marks]

(c) It is subsequently established that the mean length of adult male saltwater crocodiles exceeds that of adult female saltwater crocodiles by 1.85 metres.

Determine the power for a test of the hypothesis in part **(a)** at the 5% level of significance, based upon random samples of 40 adult male saltwater crocodiles and 30 adult female saltwater crocodiles.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 7**

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END OF QUESTIONS

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